

DTIC FILE COPY

2

AD

TECHNICAL REPORT ARCCB-TR-90025

# FATIGUE OF COMPOUND CYLINDERS

AD-A227 323

DTIC  
ELECTE  
OCT 11 1990  
D D

J. A. KAPP

P. S. J. CROFTON

AUGUST 1990



US ARMY ARMAMENT RESEARCH,  
DEVELOPMENT AND ENGINEERING CENTER  
CLOSE COMBAT ARMAMENTS CENTER  
BENÉT LABORATORIES  
WATERVLIET, N.Y. 12189-4050



APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

90 10 10 120

#### DISCLAIMER

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

The use of trade name(s) and/or manufacturer(s) does not constitute an official indorsement or approval.

#### DESTRUCTION NOTICE

For classified documents, follow the procedures in DoD 5200.22-M, Industrial Security Manual, Section II-19 or DoD 5200.1-R, Information Security Program Regulation, Chapter IX.

For unclassified, limited documents, destroy by any method that will prevent disclosure of contents or reconstruction of the document.

For unclassified, unlimited documents, destroy when the report is no longer needed. Do not return it to the originator.

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ARCCB-TR-90025	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) FATIGUE OF COMPOUND CYLINDERS		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) J.A. KAPP AND P.S.J. CROFTON (See Reverse)		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army ARDEC Benet Laboratories, SMCAR-CCB-TL Watervliet, NY 12189-4050		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AMCNS No. 6111.02.H610.011 PRON No. 1A92Z9CANMSC
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army ARDEC Close Combat Armaments Center Picatinny Arsenal, NJ 07806-5000		12. REPORT DATE August 1990
		13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) High Pressure Vessels Fatigue Design Optimization		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Compound cylinders have historically been designed such that the maximum shear stress is equal in each cylinder sector. This is the optimum condition for yielding in the cylinder since all sectors yield at the same pressure. If compound cylinders are subject to fatigue, this is not the case. A better design criterion is to equate the maximum tensile normal stress in each sector, since the maximum tensile stress controls fatigue crack propagation. This assumes that most of the life is spent in crack propagation. The derivation (CONT'D ON REVERSE)		

## 7. AUTHORS (CONT'D)

P.S.J. Crofton  
Department of Mechanical Engineering  
Imperial College of Science and Technology  
London SW7 2BX  
United Kingdom

## 20. ABSTRACT (CONT'D)

of the geometric conditions resulting in equal tensile stresses in each sector is presented in this report. The geometric optimization of the general cylinder with  $n$  sectors is very complex and is a function of the internal and external pressures and the allowable stress, unlike the optimization that assumes that the maximum shear stress dominates. There is one case for which the two methods can be explicitly compared, i.e., a cylinder with only two sectors and no external pressure. This case is studied in detail. The liners of these cylinders are much thinner when the maximum shear stress is assumed than when the maximum shear stress is equalized. To demonstrate that the maximum normal stress is a superior technique, several cylinders were designed to have a constant fatigue life by using both this method and the maximum shear stress method. In all cases, the cylinders had fatigue lives that were at least ten times the design life, the number of cycles at which loading was stopped. However, those cylinders designed according to the maximum normal stress theory showed evidence of crack initiation in many cases. Although both methods give very conservative results, the maximum shear stress theory is considered overly conservative, since it permits much lower internal pressures than the maximum normal stress theory.

UNCLASSIFIED

# TABLE OF CONTENTS

	Page
NOMENCLATURE .....	ii
INTRODUCTION .....	1
ANALYSIS .....	3
The Outermost Jackets .....	3
The Liner and Inner Jackets .....	5
The Special Case of Two Sectors and No External Pressure .....	7
EXPERIMENTAL PROCEDURE .....	9
RESULTS AND DISCUSSION .....	10
REFERENCES .....	12

## TABLES

I. DESIGN PARAMETERS FOR EXPERIMENTAL STUDY - OVERALL RADIUS RATIO $X = 2.25$ .....	9
II. MEASURED FATIGUE LIVES COMPARED WITH PREDICTED LIVES .....	10

## LIST OF ILLUSTRATIONS

1. Schematic of a sectored cylinder .....	13
2. Comparison of ideal liner matings for two theories .....	14
3. Comparison of allowable pressures for two theories .....	14



Date By Dist. to Approved Serial	
A-1	

## NOMENCLATURE

$c$	crack length
$C_1$	material constant
$D$	the denominator of a derivative
$dc/dN$	fatigue crack growth rate
$ID$	inside diameter
$k_i$	radius ratio of the $i^{th}$ cylinder sector ( $k_i=r_{i+1}/r_i$ )
$K_{Ic}$	fracture toughness
$K_{max}$	maximum stress intensity factor
$K_{min}$	minimum stress intensity factor
$m$	material constant
$n$	total number of sectors in a compound cylinder
$N$	number of fatigue cycles
$OD$	outside diameter
$p_i$	pressure that acts as $r_i$ when the cylinder is pressurized
$r_i$	$i^{th}$ interface radius
$S_{allow}$	allowable stress intensity
$X$	overall radius ratio of a compound cylinder ( $X=r_{n+1}/r_1$ )
$X_i$	radius ratio of the $i+1^{th}$ and $i^{th}$ sectors ( $X_i=r_{i+2}/r_i = k_{i+1}k_i$ )
$\Delta K_{eff}$	effective stress intensity factor range
$\sigma_{max}$	maximum normal stress
$\tau_{max}$	maximum shear stress

## INTRODUCTION

Several techniques have been used to produce cylinders containing very high pressures. Among these are autofrettage, wire wrapping, and interference fitting of several sectors to produce a multilayered cylinder (ref 1). Historically, these methods have been developed to strengthen cylinders against plastic deformation due to static pressure. If the cylinder is subjected to repeated pressure, it must be designed for optimum fatigue resistance rather than for static strength. In the case of autofrettaged cylinders, the optimum condition for static loading and fatigue loading is essentially the same. This is not the case, however, when designing compound cylinders.

Many investigators have studied compound cylinders and derived the optimum geometric conditions to prevent elastic breakdown (ref 2). It was found that the sectors should be designed such that the maximum shear stress in each sector is equal. Thus, each sector should begin to yield simultaneously. If we consider fatigue failure rather than elastic breakdown, the criterion for the optimum condition would be either for fatigue crack initiation to occur simultaneously in each sector or for pre-existing cracks to grow at the same rate in each segment. For this study, we chose to optimize with respect to crack propagation, assuming that most of the life is expended in crack growth and that cracks initiate very early in the fatigue process.

The effects of mean stress on crack propagation must be taken into account to predict fatigue crack growth in multilayered cylinders, since liners are subjected to compressive residual stress and jackets are subjected to tensile residual stress due to the interference fit of manufacture. Some guidance can be taken from prior work on crack propagation in autofrettaged cylinders. Fatigue cracks initiated at the inside diameter (ID) must grow through compressive residual stresses, while cracks initiated at the outside diameter

(OD) grow through a tensile residual stress field. The ID case has been studied more fully and a crack growth model in residual stress fields based on crack closure gives adequate results (ref 3). Crack closure occurs when the applied pressure is insufficient to overcome the compressive residual stresses and open the crack. Fatigue crack growth is assumed to occur only while the crack is indeed open and the effective stress intensity factor range ( $\Delta K_{eff} = K_{max} - K_{min}$ ) which governs crack growth rate ( $dc/dN$ ) is limited to  $K_{max}$ . The power law that represents crack growth under these circumstances is represented as

$$\frac{dc}{dN} = C_1 K_{max}^m \quad (1)$$

Although few studies have been made on the tensile residual stress effects in autofrettaged cylinders, some studies have been made where several crack growth laws were used to predict crack growth through the tensile residual stress field in the outer portion of autofrettaged cylinders (ref 4). Tensile residual stresses increase the rate of crack growth rather than retard it as with compressive residual stresses. The application of the model represented in Eq. (1) did not give the best characterization of the crack growth behavior, but it was determined to be the most conservative of those studied. It is not unreasonable to use Eq. (1) for crack growth through tensile residual stress fields as well as compressive residual stress fields.

The optimization of sectorized cylinders with respect to fatigue crack growth (accounting for residual stress effects) then involves equating the maximum stress intensity factor that occurs during the pressure cycling in each of the sectors in the multilayered cylinder. This has the added advantage of optimizing with respect to fracture, since final fracture occurs when  $K_{max}$  is equal to the fracture toughness ( $K_{IC}$ ) of the material from which the cylinder is made.



Therefore, we should be able to design a sectorized cylinder in which all the sectors fail at the same number of pressure cycles.

To actually ensure that  $K_{\max}$  is equal in each cylinder is an involved problem. Stress intensity factor calibrations for thick-walled cylinders with various radius ratios ( $k$ ) must be known. This is further complicated by the fact that the liner will have pressure in the crack, while the jackets will not. To simplify matters, we assume that the maximum stress intensity factor is proportional to the maximum normal stress ( $K_{\max} \propto \sigma_{\max}$ ). Then we can proceed with the derivation of optimum geometric designs for various configurations.

## ANALYSIS

### The Outermost Jackets

Figure 1 shows the schematic of a compound cylinder containing small fatigue cracks of equal length in each sector. The variables used are  $r_i$  which denotes the  $i^{\text{th}}$  interface radius,  $p_i$  which is the pressure that acts at  $r_i$  when the internal pressure is applied, and  $k_i$  which is the radius ratio of the  $i^{\text{th}}$  sector ( $k_i = r_{i+1}/r_i$ ). We consider the two sectors  $i$  and  $i+1$  separately from the remainder of the total cylinder and assume that the local radius ratio of this pair is fixed and given the value  $X_i$ , ( $X_i = r_{i+2}/r_i$ ). The cylinder should be constructed so that the maximum normal stress in each sector is equal. The maximum stress is the hoop stress at the inner radius of each sector and is given by the Lamé solution (ref 2).

At  $r_i$ :

$$\sigma_{\max} = \frac{k_i^2}{k_i^2 - 1} (-2p_{i+1} + p_i \frac{k_i^2 + 1}{k_i^2}) \quad (2)$$

At  $r_{i+1}$ :

$$\sigma_{\max} = \frac{k_{i+1}^2}{k_{i+1}^2 - 1} (-2p_{i+2} + p_{i+1} \frac{k_{i+1}^2 + 1}{k_{i+1}^2}) \quad (3)$$

To determine the optimum geometric conditions for equal stresses, we solve both Eqs. (2) and (3) for the pressure acting at the interface between the  $i$ th and the  $i+1$ th sectors ( $p_{i+1}$ ). Since  $p_{i+1}$  is the same in both equations, we equate expressions for  $p_{i+1}$  and solve the resulting equation for  $\sigma_{\max}$

$$\sigma_{\max} = p_i \frac{k_i^4 + (X_i^2 + 1)^2 k_i^2 + X_i^2}{-k_i^4 + (3X_i^2 - 1)k_i^2 - X_i^2} - 2p_{i+2} \frac{4X_i^2 k_i^2}{-k_i^4 + (3X_i^2 - 1)k_i^2 - X_i^2} \quad (4)$$

The optimum mating of  $k_i$  and  $k_{i+1}$  for a fixed  $X_i$  occurs when we set either  $d\sigma_{\max}/dk_i$  or  $d\sigma_{\max}/dk_{i+1}$  equal to zero. Equation (4) was derived such that the differentiation is with respect to  $k_i$ . It could also have been developed for differentiation with respect to  $k_{i+1}$ , however, the end result would be the same since  $k_i$  and  $k_{i+1}$  are related to  $X_i$  by  $X_i = k_i k_{i+1}$ . Since Eq. (4) is a fraction, its derivatives are also fractions. The condition for the derivative being equal to zero is that the numerator of the derivative is equal to zero, namely

$$D \frac{d\sigma_{\max}}{dk_i} = 8X_i^2 k_i (k_i^4 p_i + X_i^2 (p_{i+2} - p_i)) = 0 \quad (5)$$

where  $D$  is a function of  $k_i$  and the denominator of the derivative of Eq. (4). It is clear that the ideal mating of these two adjacent sectors is not solely a function of geometry, but is also a function of the boundary conditions  $p_i$  and  $p_{i+2}$ . The roots of Eq. (5) can be determined by the change of variable  $k_i' = k_i^2$ . This reduces the quartic to a quadratic whose roots are determined by applying the quadratic formula. The optimum value of  $k_i$  is taken as the square root of the positive root of  $k_i'$

$$k_i = \sqrt{X_i} \left( 1 - \frac{p_{i+2}}{p_i} \right)^{1/2} \quad (6)$$

If the pressure  $p_{i+2}$  is equal to zero, Eq. (6) reduces to the same solution that results when the geometry is optimized by equating the maximum shear stress in each sector (ref 2), namely,  $k_i = k_{i+1} = \sqrt{X_i}$ . This is the case for the two outermost sectors when the total cylinder is subjected to only internal pressure and there are more than two sectors. If there are only two sectors, the following optimization must be used that includes the effects of internal pressure in the crack.

### The Liner and Inner Jackets

The above derivation assumes that no pressure acts in any crack. The innermost liner is subjected to the applied fluid pressure. If the crack is open, pressure will infiltrate the crack and act on the crack faces. The resulting maximum stress intensity factor will be proportional to the maximum normal stress from the Lamé solution plus the internal pressure. A special derivation must be followed to account for this. The stresses that are to be equated now are

At  $r_1$ :

$$\sigma_{\max} = \frac{k_1^2}{k_1^2 - 1} (-2p_2 + 2p_1) \quad (7)$$

At  $r_2$ :

$$\sigma_{\max} = \frac{k_2^2}{k_2^2 - 1} (-2p_3 + p_2 \frac{k_2^2 + 1}{k_2^2}) \quad (8)$$

The optimum value of  $k_1$  is determined by eliminating the intermediate pressure  $p_2$  and solving for  $\sigma_{\max}$ , as above. In this instance, the resulting equation as a function of  $k_1$  and  $X_1$  (which is again assumed to be fixed) is

$$\sigma_{\max} = p_1 \frac{2X_1^2 k_1^2 + 2k_1^4}{-k_1^4 + (3X_1^2 - 1)k_1^2 - X_1^2} - p_3 \frac{4X_1^2 k_1^2}{-k_1^4 + (3X_1^2 - 1)k_1^2 - X_1^2} \quad (9)$$

Once more the maximum stress value is not a function of geometry alone, but is also a function of the boundary conditions  $p_1$  and  $p_3$ . The optimum geometry is found by differentiating with respect to  $k_1$  and setting the derivative equal to zero. Again, the result is a quartic

$$0 \frac{d\sigma_{\max}}{dk_1} = 4k_1 \{ [(4X_1^2 - 1)k_1^4 - 2X_1^2 k_1^2 - X_1^4] p_1 - (2X_1^2 k_1^4 - 2X_1^4) p_3 \} \quad (10)$$

The optimum value of  $k_1$  is determined by reducing the quartic to a quadratic and using the quadratic formula in the same manner as outlined above.

$$k_1^2 = \frac{X_1^2 \pm X_1^2 \sqrt{1 + [4X_1^2 - 1 + 2(p_3/p_1)(1 - 10X_1^2) - (p_3/p_1)^2]}}{4X_1^2 - 1 - 2X_1^2(p_3/p_1)} \quad (11)$$

Equation (11) is a complicated expression showing that the optimum geometry is a function of the pressures  $p_1$  and  $p_3$ .

To optimize the mating of the general multilayered cylinder with  $n$  sectors and an overall radius ratio of  $X = k_1 \cdot k_2 \cdots k_{n-1} \cdot k_n$  subjected to internal pressure  $p_1$  and external pressure  $p_{n+1}$ , is indeed a difficult process. The solution is obtained by starting at the OD or the ID where the value of  $p_{n+1}$  or  $p_1$  is known and working either in toward the ID or out toward the OD, whichever the case may be. For example, if we start at the OD and  $p_{n+1}$  is known, we can use Eq. (4) to solve for the ratio  $p_{n+1}/p_{n-1}$  in terms of  $\sigma_{\max}$ , the local combined radius ratio  $X_{n-1}$ , and  $k_{n-1}$  which can be substituted into Eq. (6) to give an expression for the optimum  $k_{n-1}$  in terms of  $X_{n-1}$ . This can then be substituted into either Eq. (2) or (3) to determine an implicit expression for  $p_{n-1}$ . Repeating the procedure for the mating pair of  $k_{n-2}$  and  $k_{n-1}$  and so on until we get to the pair  $k_2$  and  $k_1$ , where we must use Eqs. (7), (8), (9), and (11) rather than Eqs. (2), (3), (4), and (6), we generate a system of  $n-1$  nonlinear equations involving  $2n-1$  variables:  $k_1, \dots, k_n$  and  $X_1, \dots, X_{n-1}$ . We add another  $n-1$

equation by realizing that  $X_i = k_i k_{i+1}$  and the final equation is the design parameter for  $X$

$$X = k_1 \cdot k_2 \cdots k_i \cdots k_{n-1} \cdot k_n \quad (12)$$

Thus, in theory it is possible to optimize the general multilayered cylinder given the requirements:  $X$ ,  $p_1$ ,  $n$ ,  $p_{n+1}$ , and an allowable  $\sigma_{\max}$ . This is not a trivial task as some technique is required to solve systems of nonlinear simultaneous equations. The actual solution method depends on the particular case and is left to the reader. The important thing to note about the results obtained is that unlike the case for optimizing with respect to maximum shear stress, there is no single geometric ideal mating based on  $X$  alone. The optimum mating of the general compound cylinder for fatigue application is a function of the allowable  $\sigma_{\max}$  and the design pressure. In other words, for a given  $X$ , the optimum design for an allowable stress of 500 MPa and 20 kbar internal pressure would be different than the design for an allowable stress of 600 MPa and 15 kbar internal pressure. If the cylinder were designed such that the optimum mating for either of the two design conditions stated above was based on the shear stress criterion, the geometry of each sector  $k_1 \cdots k_n$  would be the same. Only the interface pressures would be different to meet the requirements.

#### The Special Case of Two Sectors and No External Pressure

There is one case that can be optimized explicitly and is a function of geometry alone. This occurs if  $n = 2$  and  $p_3 = 0$ . We use this case to compare the optimization method developed here and the more traditional approach of equating maximum shear stresses. The ideal mating for a required radius ratio  $X$  and two sectors is given by

$$k_1 = \sqrt{X} \quad (13)$$

$$k_1^2 = \frac{X^2 + 2X^3}{4X^2 - 1} \quad (14)$$

Equation (13) is from the shear stress criterion (ref 2) and Eq. (14) is Eq. (11) simplified by setting  $p_3$  equal to zero. It is clear that geometric optima based on the two different criteria are substantially different. The two equations are plotted in Figure 2 for a clearer comparison. The liner is always thinner when we use the maximum normal stress criterion because of the necessity for a larger compressive residual stress at the bore than in the case of equal shear stresses.

A more interesting comparison is between the allowable stresses. Suppose we design a cylinder for a given fatigue life and find that the allowable stress intensity is  $S_{allow}$ . Using the normal stress criterion, the design pressure is found from  $\sigma_{max} = S_{allow}$ , and using the shear stress criterion, the design pressure is found from  $2\tau_{max} = S_{allow}$ . The equations for the allowable pressure are

Shear stress criterion:

$$\frac{p}{S_{allow}} = \frac{X-1}{X} \quad (15)$$

Normal stress criterion:

$$\frac{p}{S_{allow}} = \frac{-k_1^4 + (3X^2-1)k_1^2 - X^2}{2X^2k_1^2 + 2k_1^4} \quad (16)$$

Equations (15) and (16) are plotted in Figure 3. This graph shows that for a given  $X$  and  $S_{allow}$ , the normal stress criterion will always allow for a greater design pressure than if the shear stress criterion were applied. This demonstrates that if the initial assumptions are correct, and the maximum normal stress dominates the fatigue behavior of compound cylinders, the use of the maximum shear stress criterion is not the optimum procedure. Furthermore, if the cylinder were designed according to the shear stress criterion, those cylinders would be quite overdesigned.

## EXPERIMENTAL PROCEDURE

To demonstrate that designing sectorized cylinders for fatigue application using the shear stress optimization is overly conservative, a short experimental program was devised. It was decided to design cylinders for a given fixed  $X$  of 2.25 and a given fatigue life of either 20,000 cycles or 100,000 cycles. The material used was ASTM A-723 Grade 3, Class 1 pressure vessel steel. This steel has a minimum tensile strength of 795 MPa. Using the fatigue ( $S - N$ ) curves for this material that appear in the ASME Pressure Vessel Code, Section VIII, Division 2, we find that the allowable stress intensity amplitude is 261 MPa for a life of 20,000 cycles and 174 MPa for a life of 100,000 cycles.

Using Eqs. (15) and (16) we can determine the applied internal pressure that results in the allowable stress intensity amplitudes. These are presented in Table I along with the other pertinent data necessary to produce two sets of specimens (one each for the two different design methods). The additional data include liner radius ratios  $k_1$  and interference pressures for each design. For each condition, two specimens were manufactured and fatigue tested using Bristol high pressure fatigue machines.

TABLE I. DESIGN PARAMETERS FOR THE EXPERIMENTAL STUDY - OVERALL RADIUS RATIO  $X = 2.25$

Design Life (Cycles)	Optimization Criterion	$k_1$	Interference Pressure (MPa)	Internal Pressure (MPa)
20,000	Shear Stress	1.500	60.7	284.3
100,000	Shear Stress	1.500	35.6	190.3
20,000	Normal Stress	1.203	59.4	370.0
100,000	Normal Stress	1.203	39.8	247.7

## RESULTS AND DISCUSSION

The measured fatigue lives of the eight specimens tested are presented in Table II. Pressure cycling of any specimen was suspended once it achieved a life equal to ten times the design life. As evident in Table II, every specimen

TABLE II. MEASURED FATIGUE LIVES COMPARED WITH PREDICTED LIVES

Specimen No.	Optimization Criterion	Design Life (Cycles)	Actual Life (Cycles)
S20-1	Shear Stress	20,000	>200,000
S20-2	Shear Stress	20,000	>200,000
S100-1	Shear Stress	100,000	>1,000,000
S100-2	Shear Stress	100,000	>1,000,000
N20-1	Normal Stress	20,000	>200,000
N20-2	Normal Stress	20,000	>200,000
N100-1	Normal Stress	100,000	>1,000,000
N100-2	Normal Stress	100,000	>1,000,000

tested met this criterion. Some specimens designed according to the maximum normal stress criterion had cracked liners, whereas no specimen designed according to the maximum shear stress criterion showed any evidence of crack initiation when cycling was stopped. Although some liners were cracked, there were no gross, through-thickness failures. Therefore, all specimens had a safety factor of ten on design fatigue life. At first glance, this may seem to prove little, except that the maximum shear stress criterion and the maximum normal stress criterion are both very conservative. The significant result from these tests is that the maximum shear stress criterion is clearly more conservative than the maximum normal stress criterion. For a fixed overall radius ratio  $X$  and allowable stress intensity  $S_{allow}$ , the maximum normal stress criterion permits substantially more pressure to be safely contained. In the



particular case of  $X = 2.25$  studied here, the permitted pressure is 30 percent greater using the maximum normal stress than the maximum shear stress. On the other hand, if we design a cylinder for a given internal pressure and allowable stress intensity, then the two theories will allow very different cylinders as a result. Let us assume that the ratio of internal pressure to allowable stress intensity is 0.6. From Figure 3, we need a cylinder with  $X = 2.5$  using the maximum shear stress theory, while using the maximum normal stress theory a cylinder with an overall radius ratio of  $X = 1.9$  is necessary. Since this is the case in many instances, rather than designing to a fatigue life requirement, using the maximum normal stress theory developed herein is more advantageous since much less material is required.

## REFERENCES

1. T.E. Davidson and D.P. Kendall, "The Design of Pressure Vessels for Very High Pressure Operation," in: The Mechanical Behavior of Materials Under Pressure, (H.L.D. Pugh, ed.), Elsevier Publishing Company, Amsterdam, 1970.
2. W.R.D. Manning and S. Labrow, in: High Pressure Engineering, Chemical and Process Engineering Series, (I.L. Hepner, series ed.), Leonard Hill, London, 1971, pp. 52-80.
3. J.H. Underwood and J.F. Throop, "Surface Crack K-Estimates and Fatigue Life Calculations in Cannon Tubes," ASTM STP 687, American Society for Testing and Materials, Philadelphia, PA, 1979, pp. 195-210; also ARRADCOM Technical Report ARLCB-TR-80021, Benet Weapons Laboratory, Watervliet, NY, June 1980.
4. J.A. Kapp and S.L. Pu, "Failure Design of Thick-Walled Cylinders Considering the OD as a Failure Initiation Site," ARRADCOM Technical Report ARLCB-TR-82029, Benet Weapons Laboratory, Watervliet, NY, September 1982.

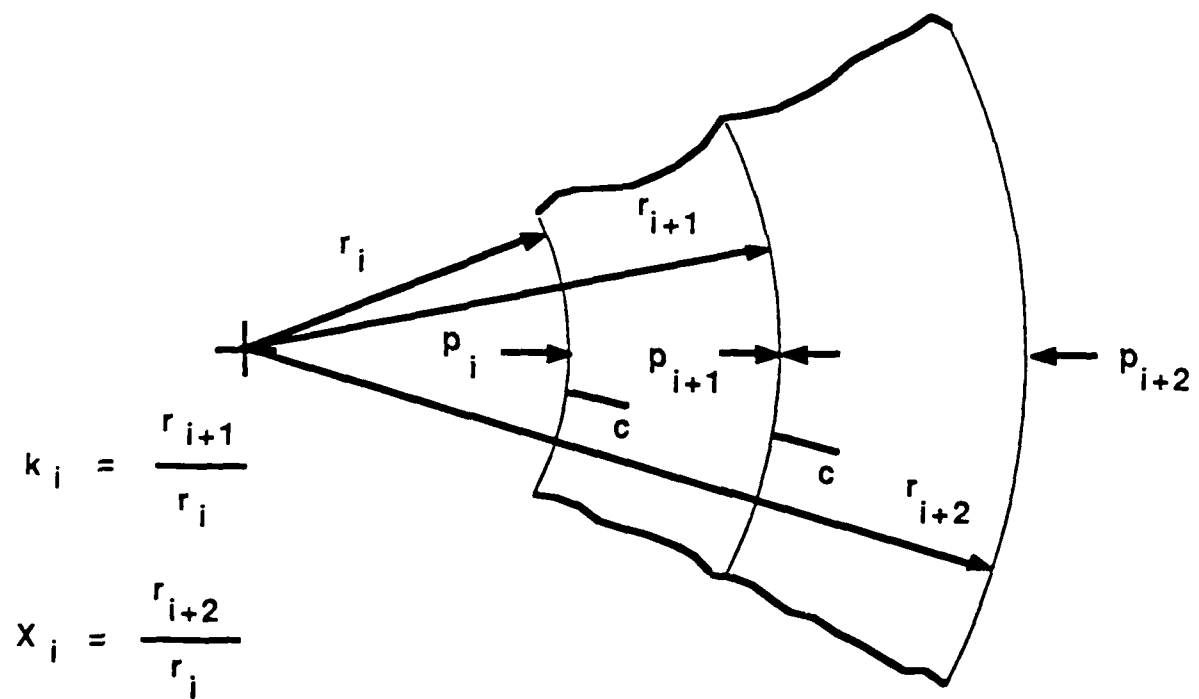


Figure 1. Schematic of a sectored cylinder.

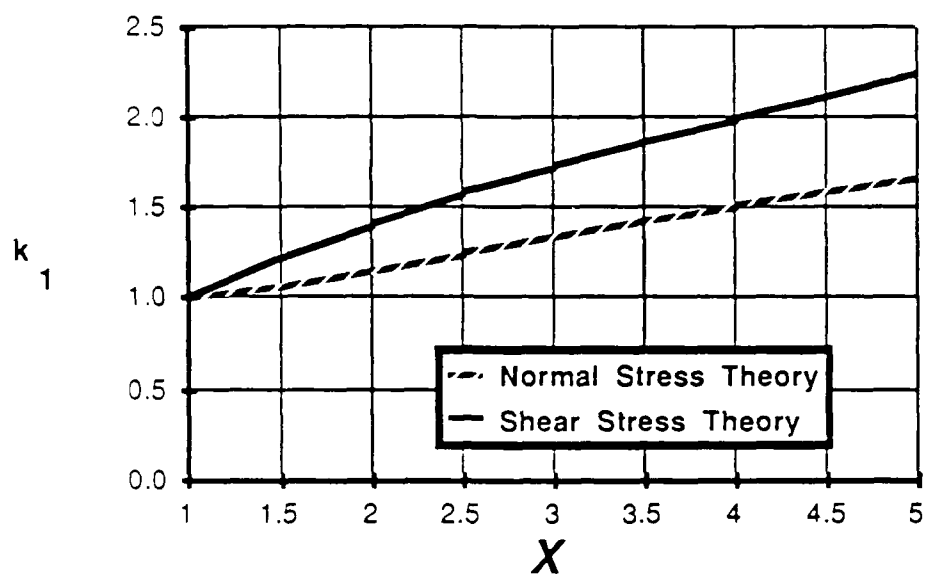


Figure 2. Comparison of ideal liner matings for two theories.

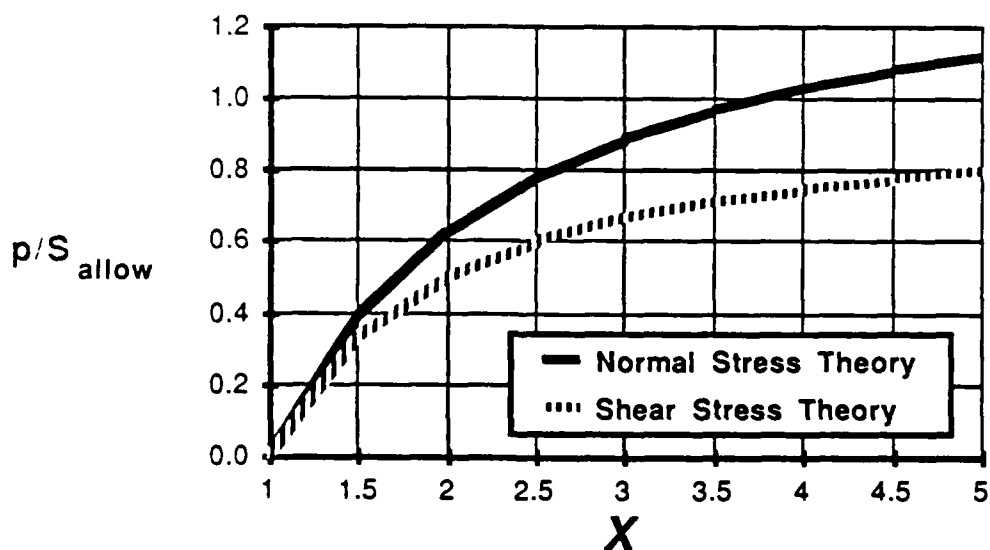


Figure 3. Comparison of allowable pressures for two theories.

# TECHNICAL REPORT INTERNAL DISTRIBUTION LIST

	NO. OF COPIES
CHIEF, DEVELOPMENT ENGINEERING DIVISION	
ATTN: SMCAR-CCB-D	1
-DA	1
-DC	1
-DI	1
-DP	1
-DR	1
-DS (SYSTEMS)	1
CHIEF, ENGINEERING SUPPORT DIVISION	
ATTN: SMCAR-CCB-S	1
-SE	1
CHIEF, RESEARCH DIVISION	
ATTN: SMCAR-CCB-R	2
-RA	1
-RE	1
-RM	1
-RP	1
-RT	1
TECHNICAL LIBRARY	5
ATTN: SMCAR-CCB-TL	
TECHNICAL PUBLICATIONS & EDITING SECTION	3
ATTN: SMCAR-CCB-TL	
DIRECTOR, OPERATIONS DIRECTORATE	1
ATTN: SMCWV-OD	
DIRECTOR, PROCUREMENT DIRECTORATE	1
ATTN: SMCWV-PP	
DIRECTOR, PRODUCT ASSURANCE DIRECTORATE	1
ATTN: SMCWV-QA	

NOTE: PLEASE NOTIFY DIRECTOR, BENET LABORATORIES, ATTN: SMCAR-CCB-TL, OF ANY ADDRESS CHANGES.

# TECHNICAL REPORT EXTERNAL DISTRIBUTION LIST

	NO. OF COPIES		NO. OF COPIES
ASST SEC OF THE ARMY RESEARCH AND DEVELOPMENT ATTN: DEPT FOR SCI AND TECH THE PENTAGON WASHINGTON, D.C. 20310-0103	1	COMMANDER ROCK ISLAND ARSENAL ATTN: SMCRI-ENM ROCK ISLAND, IL 61299-5000	1
ADMINISTRATOR DEFENSE TECHNICAL INFO CENTER ATTN: DTIC-FDAC CAMERON STATION ALEXANDRIA, VA 22304-6145	12	DIRECTOR US ARMY INDUSTRIAL BASE ENGR ACTV ATTN: AMXIB-P ROCK ISLAND, IL 61299-7260	1
COMMANDER US ARMY ARDEC ATTN: SMCAR-AEE	1	COMMANDER US ARMY TANK-AUTMV R&D COMMAND ATTN: AMSTA-DDL (TECH LIB) WARREN, MI 48397-5000	1
SMCAR-AES, BLDG. 321	1	COMMANDER	
SMCAR-AET-O, BLDG. 351N	1	US MILITARY ACADEMY	1
SMCAR-CC	1	ATTN: DEPARTMENT OF MECHANICS	
SMCAR-CCP-A	1	WEST POINT, NY 10996-1792	
SMCAR-FSA	1		
SMCAR-FSM-E	1	US ARMY MISSILE COMMAND	
SMCAR-FSS-D, BLDG. 94	1	REDSTONE SCIENTIFIC INFO CTR	2
SMCAR-IMI-I (STINFO) BLDG. 59	2	ATTN: DOCUMENTS SECT, BLDG. 4484	
PICATINNY ARSENAL, NJ 07806-5000		REDSTONE ARSENAL, AL 35898-5241	
DIRECTOR US ARMY BALLISTIC RESEARCH LABORATORY ATTN: SLCBR-DD-T, BLDG. 305	1	COMMANDER US ARMY FGN SCIENCE AND TECH CTR ATTN: DRXST-SD	1
ABERDEEN PROVING GROUND, MD 21005-5066		220 7TH STREET, N.E. CHARLOTTESVILLE, VA 22901	
DIRECTOR US ARMY MATERIEL SYSTEMS ANALYSIS ACTV ATTN: AMXSY-MP	1	COMMANDER US ARMY LABCOM	
ABERDEEN PROVING GROUND, MD 21005-5071		MATERIALS TECHNOLOGY LAB ATTN: SLCMT-IML (TECH LIB)	2
COMMANDER HQ, AMCCOM		WATERTOWN, MA 02172-0001	
ATTN: AMSMC-IMP-L	1		
ROCK ISLAND, IL 61299-6000			

NOTE: PLEASE NOTIFY COMMANDER, ARMAMENT RESEARCH, DEVELOPMENT, AND ENGINEERING CENTER, US ARMY AMCCOM, ATTN: BENET LABORATORIES, SMCAR-CCB-TL, WATERVLIET, NY 12189-4050, OF ANY ADDRESS CHANGES.

TECHNICAL REPORT EXTERNAL DISTRIBUTION LIST (CONT'D)

	<u>NO. OF COPIES</u>		<u>NO. OF COPIES</u>
COMMANDER US ARMY LABCOM, ISA ATTN: SLCIS-IM-TL 2800 POWDER MILL ROAD ADELPHI, MD 20783-1145	1	COMMANDER AIR FORCE ARMAMENT LABORATORY ATTN: AFATL/MN EGLIN AFB, FL 32542-5434	1
COMMANDER US ARMY RESEARCH OFFICE ATTN: CHIEF, IPO P.O. BOX 12211 RESEARCH TRIANGLE PARK, NC 27709-2211	1	COMMANDER AIR FORCE ARMAMENT LABORATORY ATTN: AFATL/MNF EGLIN AFB, FL 32542-5434	1
DIRECTOR US NAVAL RESEARCH LAB ATTN: MATERIALS SCI & TECH DIVISION CODE 26-27 (DOC LIB) WASHINGTON, D.C. 20375	1 1	METALS AND CERAMICS INFO CTR BATTELLE COLUMBUS DIVISION 505 KING AVENUE COLUMBUS, OH 43201-2693	1

NOTE: PLEASE NOTIFY COMMANDER, ARMAMENT RESEARCH, DEVELOPMENT, AND ENGINEERING CENTER, US ARMY AMCCOM, ATTN: BENET LABORATORIES, SMCAR-CCB-TL, WATERVLIET, NY 12189-4050, OF ANY ADDRESS CHANGES.